

RAMAKRISHNA MISSION VIDYAMANDIRA
 (Residential Autonomous College affiliated to University of Calcutta)
M.A./M.Sc. FIRST SEMESTER EXAMINATION, MARCH 2022
FIRST YEAR [BATCH 2021-23]
Mathematics

Date : 28/03/2022

Time : 11.00 am – 1.00 pm

Complex Analysis

Paper : MTM CC5

Full Marks:50

Answer all the questions, maximum one can score 50.

(All notations have their usual meaning)

1. Find the number of zeroes (counting multiplicity) of $P(z) = 3z^5 + 2iz^2 + 7iz + 1$ in the annular region $\{z \in \mathbb{C} : 1 < |z| < 7\}$. Justify your answer. [0.5+5]
2. Find $\int_C f(z)dz$, where $C : |z| = 7$ (in clockwise sense) and $f(z) = \frac{z^3(1-3z)}{(1+z)(1+2z^4)}$. [4]
3. Is there a polynomial $P(z)$, such that $P(z)e^{\frac{1}{z}}$ is an entire function? Justify your answer. [0.5+4]
4. Use suitable contour to show $\int_0^\infty \cos x^2 dx = \frac{\sqrt{\pi}}{2\sqrt{2}}$. [10]
5. Find the linear fractional transformation which sends 0, 1, 2 to $-1, 0, 4$ respectively. [6]
6. Let f be continuous on the unit circle $|z| = 1$ and

$$f(e^{it}) = \frac{3 + 2 \cos t}{13 + 12 \cos t} - i \frac{2 \sin t}{13 + 12 \cos t}, \quad t \in [0, 2\pi)$$

Find a function $h(z)$ which is analytic in the unit disc and coincides with $f(z)$ on $|z| = 1$. [8]

7. If C denotes the positively oriented boundary of the square whose sides lie along the straight lines $x = \pm 2$ and $y = \pm 2$; then evaluate $\int_C f(z)dz$ where $f(z)$ are the following:

(a) $f(z) = \frac{e^{-z}}{z - \frac{\pi i}{2}}$ [4]

(b) $f(z) = \frac{\cosh z}{z^4}$ [5]

8. Expand the function $f(z) = \frac{z}{1+z^3}$

(a) in a series of positive powers of z

(b) in a series of negative powers of z .

In each case specify the region where the expression is valid. [4+4]

9. Suppose $u : \mathbb{R}^2 \rightarrow \mathbb{R}$, is a bounded harmonic function. Show that u is constant. [5]

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